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ON THE THEORY OF HIGH FREQUENCY PLASMA OSCILLATIONS

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As is known, the electrical conductivity of plasma, the time of establishing the thermal equilibrium between electrons and ions, and also the time of heating the electron component of plasma all increase greatly with temperature. Consequently, the usual Joule method of heating plasma may be difficult to apply in the region of high temperatures (above 10^{7}°K), especially if the current alone (without any additional measures) is used for confinement of the plasma. Therefore, it is of interest to study other methods of heating plasma which do not directly use Joule heat. Methods by which energy is directly supplied to the ion component between collisions of the particles are of special interest.

Some of such methods make use of ionic resonance as well as other resonance phenomena of the plasma, in an external magnetic field.

In this connection it is of interest to study systematically the high frequency properties of plasma in an external magnetic field.

This paper deals with certain data on the high frequency oscillations of plasma.

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I. Kinetic theory of oscillation of boundless plasma in a magnetic field

The high frequency properties of plasma may be studied most completely by means of the kinetic equation, in which the collision operator may be omitted. This equation for particles of sort α may be expressed as follows (1):

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}\,\frac{\partial f_\alpha}{\partial \vec{r}} + \frac{e_\alpha}{m_\alpha}\vec{E}\,\frac{\partial f_{o\alpha}}{\partial \vec{v}} \mp \omega_\alpha \frac{\partial f_\alpha}{\partial \varphi} = 0$$

where: $f_\alpha(\vec{r}, \vec{v}, t)$ is the perturbation of the distribution function from the equilibrium function, which we will denote as $f_{o\alpha}(v^2)$; e_α and m_α are the charge and the mass of sort α particles; $\omega_\alpha = |e_\alpha| H_0 / m_\alpha c$ (H_0 is the external constant magnetic field intensity);

$\vec{E}$ is the electric field intensity; the top sign is for ions, while the bottom sign is for electrons; the angles are shown in Fig. 1.

It is not difficult to see that the electric field intensity $\vec{E}$ satisfies the following equation:

$$\Delta\vec{E} - \operatorname{grad}\operatorname{div}\vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial}{\partial t}\sum_\alpha e_\alpha \int \vec{v}\, f_\alpha\, d\vec{v} \qquad 1.\ /2/$$

We will look for the quantities f_α and $\vec{E}$ in the form of plane waves

$$f_\alpha,\ \vec{E} \sim e^{i\vec{k}\vec{r} - i\omega' t}, \qquad \operatorname{Im}\omega' > 0$$

By substituting these equations into 1. /1/ and 1. /2/ we obtain the following set of equations:

$$\sum_{k=1}^{3}\left[n'^2(\varkappa_i \varkappa_k - \delta_{ik}) + \varepsilon_{ik}\right]E_k = 0 \qquad i = 1,2,3. \qquad 1./3/$$

where:

$$\varepsilon_{ik}(\omega', \vec{k}) = \delta_{ik} + \sum_\alpha \frac{4\pi i e_\alpha}{\omega' \omega_\alpha}\int v_i f'_{o\alpha}\, e^{i a_\alpha \sin\vartheta + i b_\alpha \vartheta}\int^{\vartheta} v_k\, e^{-i a_\alpha \sin\psi - i b_\alpha \psi}\, d\psi\, d\vec{v}$$

$$a_\alpha = \pm k v_\perp \sin\theta / \omega_\alpha \qquad b_\alpha = \pm(k v_3 \cos\theta - \omega')/\omega_\alpha$$

$$n' = \frac{kc}{\omega'}, \qquad \vec{\varkappa} = \frac{\vec{k}}{k}, \qquad f'_{o\alpha} = \frac{\partial f_{o\alpha}}{\partial \varepsilon}, \qquad \varepsilon = \frac{m_\alpha v^2}{2}$$

The quantities ε_{ik} form a tensor of dielectric constants. We see that ε_{ik} depends on the wave vector as well as on the frequency. In other words, there is space dispersion in the plasma as well as a time dispersion.

The following relation may be derived from equation 1./3/[3]:

$$\mathrm{Det}\left\{n'^2(\varkappa_i \varkappa_k - \delta_{ik}) + \varepsilon_{ik}\right\} \equiv A_n'^4 + Bn'^2 + C = 0$$

$$A = \varepsilon_{11} \sin^2\theta + \varepsilon_{33}\cos^2\theta + 2\varepsilon_{13} \sin\theta \cos\theta, \qquad 1./4/$$

$$B = 2(\varepsilon_{12}\varepsilon_{23} - \varepsilon_{22}\varepsilon_{23})\cos\theta\sin\theta + \varepsilon_{13}^2 - \varepsilon_{11}\varepsilon_{33} - (\varepsilon_{22}\varepsilon_{33} + \varepsilon_{23}^2) -$$

$$-(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{12}^2)\sin^2\theta \qquad C = \mathrm{Det}(\varepsilon_{ik})$$

It is used to determine the refraction index of waves propagating in plasma.

If ion motion is not accounted for in the above formulas, we obtain high frequency electron oscillations. Let us consider, first of all, such oscillations when the plasma temperature is equal to zero ($T_e = 0$).

The coefficients in dispersion equation 1./4/ do not depend on $\vec{K}$ with $T_e = 0$. Therefore, the solution to equation 1./4/ is as follows:

$$n^2_{\pm} = \frac{-B_0 \pm \sqrt{B_0^2 - 4A_0C_0}}{2B_0} \qquad 1./5/,$$

where

$$A_0 = 1 - u - v + uv\cos\theta$$

$$B_0 = u(2-v) - 2(1-v)^2 - uv\cos^2\theta,$$

$$C_0 = (1-v)^3 - u(1-v),$$

$$u = \left(\frac{\omega^2}{\omega}\right)^2, \quad v = \left(\frac{\Omega_e}{\omega}\right)^2, \qquad \Omega_e = \left(\frac{4\pi e^2 n_c}{m_e}\right)^{1/2}$$

This equation determines the refraction indices for ordinary and extraordinary waves in the "hydrodynamic" approximation.

Taking the coefficient A_0 as zero, we find the natural frequency for longitudinal oscillations of the plasma in a magnetic field with $T_e = 0$ [6]

$$\omega_{\pm}^2 = \frac{1}{2}\left(\Omega_e^2 + \omega_e^2\right) \pm \frac{1}{2}\left[\left(\Omega_e^2 + \omega_e^2\right)^2 - u\,\Omega_e^2\omega_e^2\cos^2\theta\right]^{1/2}$$

Now let us account for the thermal corrections, assuming that $\omega_c \gg \kappa S_e$, where S_e is the thermal velocity of the electrons equal to $S_e = \left(\frac{T_e}{m_e}\right)^{1/2}$. In this case the dispersion equation may be expressed as follows [4.3]:

$$A_1 n^6 + (A_0 + B_1) n^4 + (B_0 + C_1) n^2 + C_0 = 0 \qquad 1./6/$$

where

$$A_1 = -\frac{S_e^2}{c^2} v \left\{ 3\cos^4\theta (1-u) + \frac{6-3u+u^2}{(1-u)^2} \cos^2\theta \sin^2\theta + \frac{3\sin^4\theta}{1-4u} \right\}$$

$$B_1 = \frac{S_e^2}{c^2} v \left\{ \frac{2(1+u-v)}{1-u} \cos^2\theta \sin^2\theta + (1+\cos^2\theta)\left[(1-u-v)\left(3\cos^2\theta + \frac{\sin^2\theta}{1-u}\right) + \right.\right.$$

$$\left. + (1-v)\left(\frac{1+3u}{(1-u)^2} \cos^2\theta + \frac{3\sin^2\theta}{1-4u}\right)\right] + \frac{2\sin^2\theta}{1-u}\left[\frac{1+3u-v-4uv}{1-u} \cos^2\theta + \right.$$

$$\left.\left. + \frac{2(1-u)(1+2u-v)}{1-4u} \sin^2\theta \right]\right\},$$

$$C_1 = -\frac{S_e^2}{c^2} v \left\{ \frac{2(1-v)}{1-u}\left[\frac{1+3u-v-4uv}{1-u} \cos^2\theta + \frac{2(1-u)(1+2u-v)}{1-4u} \sin^2\theta \right] + \right.$$

$$\left. + \left[(1-v)^2 - u\right]\left(3\cos^2\theta + \frac{\sin^2\theta}{1-u}\right) \right\}$$

These equations have three roots n_1^2, n_2^2, n_3^2, which determine the refraction indices of the ordinary, extraordinary and plasma waves respectively. The first two roots are determined by the formula

$$n_{1,2} = n_{\pm}(1 + \delta_{\pm}) \qquad 1./7/,$$

where

$$\delta_{\pm} = -(A_1 n_{\pm}^4 + B_1 n_{\pm}^2 + C_1)(2A_0 n_{\pm}^2 + B_0)^{-1}, \quad /\delta_{\pm}/ \ll 1$$

while the refraction factor for the plasma wave is determined by the formula

$$n_3^2 = -\frac{A_0}{A_1}, \quad \frac{S}{c} \ll /A_0/ \ll 1 \qquad 1./8/.$$

If the frequency is near ω_+ or ω_-, [3]:

$$n_1^2 = -\frac{C_0}{B_0}, \quad n_{2,3}^2 = -\frac{A_0 \pm \sqrt{A_0^2 - 4A_1B_0}}{2A_1} \qquad 1./9/$$

Let us consider the case of resonance where $\omega \approx \omega_e$. Here, the refraction factors for the ordinary and extraordinary waves are determined by the formula

$$n_{1,2}^2 = n^2 \pm (1 + \Delta_\pm)$$

where

$$\Delta_\pm = i\sqrt{\frac{8}{\pi}}\,\frac{s_e \cos\theta}{c\,n_\pm V}\Big\{\Big[1 - \Big(\frac{1}{u}\sin^2\theta + \cos^2\theta\Big)v\Big]\,n_\pm^4 - \Big[(1-v)\Big(1-\frac{v}{4}\Big)(1+\cos^2\theta) + \Big(1-\frac{v}{2}\Big)\sin^2\theta\Big]\,n_\pm^2 + (1-v)\Big(1-\frac{v}{2}\Big)\Big\}\big(2\sin^2\theta\, n_\pm^2 - 2v - 2 - \sin^2\theta\big)^{-1} \qquad 1./10/.$$

We see that the resonance waves decay rapidly. The decay coefficient is of the same order as the quantity s_e/c ; it is much larger than the thermal corrections to the refraction indices for the ordinary and extraordinary waves.

If $\omega \approx m\omega_e$, where m = 2,3... (harmonic resonances), then $n_{1,2} = n_\pm + i\varkappa_\pm$ where [5]:

$$\varkappa_\pm \sim \Big(\frac{s_e}{c} n_\pm\Big)^{2m-3} \sin^{2m-2}\theta\, e^{-z^2 m}, \quad z_m = \frac{\big(1 - \frac{m\omega_e}{\omega}\big)c}{\sqrt{2}\, s_e\, n_\pm}$$

It should be noted that the decay is exponentially small far away from the resonance frequencies and is of the same order as the decay found by Landau [2]:

$$\gamma_L \sim \omega e^{-\frac{\omega^2}{2k^2 s_e^2}}$$

Let us now consider the longitudinal natural frequencies of the plasma. The electromagnetic waves in the plasma cannot be divided into strictly longitudinal and transverse waves in the presence of a magnetic field. However, for the limiting case of $n \to \infty$ the longitudinal plasma waves which satisfy the condition A = 0 can be singled out. Taking the thermal correction into account, the roots of the equation A = 0 are as follows [3, 6]:

$$\omega_{1,2}^2 = \omega_\pm^2 (1 \pm \varepsilon_\pm) \qquad |\varepsilon_\pm| \ll 1 \qquad 1./11/.$$

where

$$\varepsilon_\pm = \frac{k^2 s_e^2 V_\pm/\omega_\pm^2}{1 + V_\pm u_\pm (1-u_\pm)^{-2}\sin^2\theta}\Big\{3\cos^4\theta + \frac{6 - 3u_\pm + u_\pm^2}{(1-u_\pm)^3}\cos^2\theta\sin^2\theta + \frac{3\sin^4\theta}{(1-u_\pm)(1-4u_\pm)}\Big\}, \quad V_\pm = \Big(\frac{\Omega_e}{\omega_\pm}\Big)^2 \quad u_\pm = \Big(\frac{\omega_e}{\omega_\pm}\Big)^2$$

The decay of the plasma oscillations is of the order of γ_L.

Let us consider the case where the frequency of plasma oscillations ω_1 is close to $m\omega_e$ (for m=2,3...). If the angle between the direction in which the wave is propagated and the magnetic field is not near $\frac{\pi}{2}$, the plasma resonant oscillations will strongly decay. The decay coefficient is equal to 3

$$\gamma_m = \frac{\sqrt{\pi}\, m^4 \sin^{2m}\theta}{2^{m+3/2}\, m!/\cos^3\theta/\left\{1+m^4(m^2-1)^{-2}\, tg^2\,\Theta\right\}}\left(\frac{\kappa S_e}{\omega_e}\right)^{2m-4}\kappa S_e \qquad 1./12/.$$

Plasma waves with frequencies $m\omega_e-\varepsilon_m < \omega < m\omega_e+\varepsilon_m$ cannot be propagated perpendicularly to the magnetic field. Here, the "slot" in the frequency spectrum is determined from the equation (7.3),

$$\varepsilon_m = (m^2-1)(2^{m+1}m!)^{-1/2}\,(\kappa S_e/\omega_e)^{m-2}\,\kappa S_e \qquad 1./13/$$

We have considered above plasma oscillations without accounting for ion motion. Now we shall investigate low frequency plasma oscillations in which the ions as well as the electrons move. These oscillations are usually described by means of magneto-hydrodynamic equations. This is valid only for the case in which the frequency of oscillations is much smaller than the collision frequency ν. In reality, however, magneto-hydrodynamic waves may exist for any relationship between ω and ν. It must only be assumed that the frequency of oscillations is small as compared with the cyclotron ion frequency ω_i[8, 9].

It may be shown from equations 1./1/ and 1./2/ 10 that two magneto-hydrodynamic waves exist for the case an ordinary wave having a frequency of

$$\omega_1 = \kappa V_A \cos\Theta, \qquad V_A = (H_0^2/4\pi n_0 m_i)^{1/2} \ll c \qquad 1./14/$$

and an extraordinary wave having a frequency of

$$\omega_2 = \kappa V_A \qquad 1./15/.$$

The decay of the ordinary wave for $\theta \sim 1$ is determined by the following formulas:

$$(\gamma/\omega_1 \sim \sqrt{\frac{m_e}{m_i}}\cdot\frac{S_i}{V_A}\cdot\frac{\omega^2}{\omega_i^2}\exp(-V_A^2/2S_e^2), \quad S_i = (T_i/m_i)^{1/2} \ll V_A$$
$$(\gamma/\omega)_1 \sim \omega^2/\omega_i^2 \qquad S_i \sim V_A \qquad 1./16/.$$
$$(\gamma/\omega)_1 \sim S_i^3\,\omega^2/V_A^3\,\omega_i^3 \qquad S_i \gg V_A$$

The decay of the extraordinary wave is equal to

$$(\gamma/\omega)_2 = \sqrt{\frac{m_e}{m_i}}\ \frac{S_i}{V_A}\exp\left(-\frac{V_A^2}{2S_e^2\cos^2\theta}\right), \quad S_i \ll V_A \qquad 1./17/$$

These formulas, as well as equations 1./10/ and 1./12/, do not include the decay caused by "nearby" collisions.

Formulas 1./16/ and 1./17/ may be used if the decay is slight, i.e. $\gamma \ll \omega$. The rate of decay increases as the phase velocity falls. An ordinary magneto-hydrodynamic wave quickly decays ($\gamma_i \sim \omega_1 \sim KV_A$) if $S_i^3\omega_i^2 \gtrsim V_A^3\omega_i^2$. An extraordinary wave begins to decay quickly if the Alfven velocity V_A is comparable with the thermal velocity of the ions S_i.

Finally, let us consider low frequency longitudinal waves [13]. In the absence of a magnetic field the frequency of these waves is determined by the Tonks-Langmuire equation [11]:

$$\omega = \omega_0 = \frac{\Omega_i K a_e}{\sqrt{1+K^2 a_e^2}} \qquad 1./18/,$$

where

$$\Omega_i = (4\pi e^2 n_0/m_i)^{1/2} \qquad a_e = (T_e/4\pi e^2 n_0)^{1/2}$$

The decay is expressed by the following formula [12]:

$$\gamma = \gamma_0 = \sqrt{\frac{\pi m_e}{8 m_i}}\ \frac{\Omega_i K a_e}{(1+K^2 a_e^2)^2} \qquad 1./19/.$$

Here, T_e is assumed to be much larger than T_i.

Equations 1./18/ and 1./19/ may also be used when there is a weak magnetic field that satisfies the condition $\omega_e \ll K S_e$

If there is a strong magnetic field, where $\omega_i \gg K S_i$ two longitudinal waves may be propagated in the plasma at frequencies ω_1 and ω_2

$$\omega_{1,2}^2 = \frac{1}{2}(\omega_0^2+\omega_i^2) \pm \frac{1}{2}\left\{(\omega_0^2+\omega_i^2)^2 - 4\,\omega_0^2\omega_i\cos^2\theta\right\}^{1/2} \qquad 1./20/.$$

These waves have the following decay coefficients

$$\gamma_{1,2} = \sqrt{\frac{\pi}{8}}\cdot\frac{\omega_{1,2}^4}{K^3 a_e^3\left(\cos^3\theta/[1+tg^2\theta\,\omega_{1,2}^4(\omega_{1,2}^2-\omega_i^2)^{-2}]\,Z_e\,\Omega_i^2\right)} \qquad 1./21/.$$

In deriving equations 1./20/ and 1./21/ the following assumptions were made:

$$/\omega - \omega_i/ \gg K S_i\cos\theta \qquad \omega \gg K S_i\cos\theta, \qquad \omega \ll K S_e\cos\theta$$

At intermediate magnetic field intensities, where $\omega_e \gg k S_e$, $\omega_i \ll c k S_i$ and θ is not close to $\frac{\pi}{2}$, the frequency is obtained from equation 1./18/, and the decay is equal to

$$\gamma = \frac{\gamma_0}{|\cos\theta|} \qquad 1./22/.$$

2. Wave guide and resonance properties of a plasma cylinder in a longitudinal magnetic field

In order to ascertain the possibility of using high frequency heating for plasma, let us look into the wave guide and resonance properties of a plasma cylinder, which is located in a magnetic field directed along the cylinder axis. It is necessary to consider wave propagation in bounded plasma since the dispersive properties and the electromagnetic field distribution may differ markedly in unbounded and bounded plasma.

Kinetic theory gives a full picture of wave propagation, however, the basic features of the processes that are of interest to us can be found from a simpler set of equations, i.e. two-component hydrodynamic equations:

$$m_e \frac{d\vec{v}_e}{dt} = -e\vec{E} - \frac{e}{c}\left[\vec{v}_e, \vec{H}_0\right] + m_e\nu\left(\vec{v}_i - \vec{v}_e\right)$$

$$m_i \frac{d\vec{v}_i}{dt} = e\vec{E} + \frac{e}{c}\left[\vec{v}_i, \vec{H}_0\right] - m_e\nu\left(\vec{v}_i - \vec{v}_e\right) \qquad 2./1/,$$

$$\operatorname{rot}\vec{H} = \frac{4\pi}{c} en\left(\vec{v}_i - \vec{v}_e\right) + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \qquad \operatorname{rot}\vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t}$$

where $\vec{v}_e$ and $\vec{v}_i$ are the electron and ion velocities;

m_e and m_i are their masses;

n is the equilibrium density of the plasma;

ν is the effective collision frequency;

$\vec{H}_0$ is the intensity of the external magnetic field ($\vec{H}_0 \parallel z$).

Assuming that all the quantities are proportional to expi $(k_z z - \omega t)$, the following equations may be derived for determining the longitudinal components of the electric and magnetic fields of axially symmetrical waves:

$$\Delta_\perp^2 E_z + 2p\,\Delta_\perp E_z + q E_z = 0, \qquad \Delta_\perp^2 H_z + 2p\,\Delta_\perp H_z + q H_z = 0 \qquad 2./2/,$$

where

$$\Delta_\perp = \frac{1}{\rho}\frac{d}{d\rho}\,\rho\frac{d}{d\rho}$$

$$p = \frac{1}{2\varepsilon_1}\left[(\varepsilon_1^2 - \varepsilon_2^2 + \varepsilon_1\varepsilon_3)k^2 - (\varepsilon_1 + \varepsilon_3)k_3^2\right],$$

$$q = \frac{\varepsilon_3}{\varepsilon_1}\delta, \qquad \delta = \left[(\varepsilon_1 - \varepsilon_2)k^2 - k_3^2\right]\left[(\varepsilon_1 + \varepsilon_2)k^2 - k_3^2\right],$$

$$\varepsilon_1 = 1 + \frac{\Omega_e^2(1+\mu)\left[\omega^2 - \omega_e\omega_i + i\omega\nu(1+\mu)\right]}{\omega^2\omega_e^2(1-\mu)^2 - \left[\omega^2 - \omega_e\omega_i + i\omega\nu(1+\mu)\right]^2}$$

$$\varepsilon_2 = \frac{\omega\omega_e\Omega_e^2(\mu^2-1)}{\omega^2\omega_e^2(1-\mu)^2 - \left[\omega^2 - \omega_e\omega_i + i\omega\nu(1+\mu)\right]^2}$$

$$\varepsilon_3 = 1 - \frac{\Omega_e^2(1+\mu)}{\omega\left[\omega + i\nu(1+\mu)\right]}, \qquad \omega_e = \frac{eH_0}{m_e c}, \qquad \omega_i = \frac{eH_0}{m_i c}$$

$$\Omega_e^2 = \frac{4\pi e^2 n}{m_e}, \qquad \mu = \frac{m_e}{m_i}$$

The quantities $\varepsilon_1\ \varepsilon_2\ \varepsilon_3$ form a tensor of dielectric constants

$$\varepsilon_{ik} = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}$$

The solution to equation 2./2/, regular at the point is as follows:

$$E_z = AJ_0(k_1\rho) + BJ_0(k_2\rho) \qquad 2./3/$$

where

$$k_1^2 = p + \sqrt{p^2 - q}$$
$$k_2^2 = p - \sqrt{p^2 - q}$$

The remaining components of the fields inside the plasma cylinder are determined from Maxwell's equations:

$$E_\rho = -\frac{ik_1\delta^{-1}}{k_3}\mathcal{G}(AJ_1(k_1\rho) - \frac{ik_2\delta^{-1}}{k^3}RBJ_1(k_2\rho),$$

$$E_\varphi = \frac{k_1\delta^{-1}}{\varepsilon_2 k_3}QAJ_1(k_1\rho) + \frac{k_2\delta^{-1}}{\varepsilon_2 k_3}SBJ_1(k_2\rho),$$

$$H_z = -\frac{i\varepsilon_1 M}{\varepsilon_2 k k_3}AJ_0(k_1\rho) - \frac{i\varepsilon_1 N}{\varepsilon_2 k k_3}BJ_0(k_2\rho), \qquad 2./3/a,$$

$$H_\rho = -\frac{k_1\delta^{-1}}{\varepsilon_2 k}QAJ_1(k_1\rho) - \frac{k_2\delta^{-1}}{\varepsilon_2 k}SBJ_1(k_2\rho)$$

$$H_\varphi = -\frac{ik\varepsilon_3}{k_1}AJ_1(k_1\rho) - \frac{ik\varepsilon_3}{k_2}BJ_1(k_2\rho)$$

where

$$M = \frac{\varepsilon_3}{\varepsilon_1}\left(\varepsilon_1 k^2 - k_3^2\right) - k_\perp^2 \qquad N = \frac{\varepsilon_3}{\varepsilon_1}\left(\varepsilon_1 k^2 - k_3^2\right) - k_2^2$$

$$H = k_3^2\left(\varepsilon_1 k^2 - k_3^2\right) + \varepsilon_1 k^2 M$$

$$R = k_3^2\left(\varepsilon_1 k^2 - k_3^2\right) + \varepsilon_1 k^2 N$$

$$Q = \varepsilon_2^2 k^2 k_3^2 + \varepsilon_1\left(\varepsilon_1 k^2 - k_3^2\right) M$$

$$S = \varepsilon_2^2 k^2 k_3^2 + \varepsilon_1\left(\varepsilon_1 k^2 - \varepsilon_3^2\right) N$$

The following dispersion equation is evolved after making use of the boundary conditions at the surface of the plasma cylinder:

$$\frac{\varepsilon_3}{k_1 k_2}\cdot\frac{J_1(k_1R_0)J_1(k_2R_0)}{J_0(k_1R_0)J_0(k_2R_0)} + \frac{1}{\tilde{\varkappa}^2}\cdot\frac{K_1^2(\tilde{\varkappa}R_0)}{K_0^2(\tilde{\varkappa}R_0)} +$$

$$+ \frac{\varepsilon_3\left[\left(\varepsilon_1 k^2 - k_3^2\right)\left(\varepsilon_1 + 1\right) - \varepsilon_2^2 k^2\right]}{2\varepsilon_1 k_1^2 k_2^2}\cdot\frac{1}{\tilde{\varkappa}}\,\frac{K_1(\tilde{\varkappa}R_0)}{K_0(\tilde{\varkappa}R_0)}\left[k_1\frac{J_1(k_1R_0)}{J_0(k_1R_0)} + k_2\frac{J_1(k_2R_0)}{J_0(k_2R_0)}\right] -$$

$$- \frac{\varepsilon_3\left\{(\varepsilon_1-1)(\varepsilon_1-\varepsilon_3)(\varepsilon_1 k^2 - k_3^2) + \varepsilon_2^2 k^2\left[k^2\left(\varepsilon_2^2 + \varepsilon_1\varepsilon_3 - 2\varepsilon_1^2 + \varepsilon_1\right) + k_3^2\left(1 + 2\varepsilon_1 + \varepsilon_3\right)\right]\right\}}{2\varepsilon_\perp^2 k_\perp^2 k_2^2\left(k_1^2 - k_2^2\right)}\times$$

$$\times\frac{1}{\tilde{\varkappa}}\cdot\frac{K_1(\tilde{\varkappa}R_0)}{K_0(\tilde{\varkappa}R_0)}\cdot\left[k_1\frac{J_1(k_1R_0)}{J_0(k_1R_0)} - k_2\frac{J_1(k_2R_0)}{J_0(k_2R_0)}\right] = 0$$

where $\tilde{\varkappa}^2 = k_3^2 - k^2$, and R_0 is the radius of the plasma cylinder.

The electron and ion velocities for the general case are as follows:

$$v_{e\rho} = \frac{e}{m_e\Delta_1}\left\{i\omega\left[\omega^2 - \omega_i^2 + i\omega\nu^*\right]E_\rho + \omega_e\left[\omega^2 - \omega_i + i\omega\nu^*\mu\right]E_\varphi\right\},$$

$$v_{e\varphi} = \frac{e}{m_e\Delta_1}\left\{-\omega_e\left[\omega^2 - \omega_i^2 + i\omega\mu\nu^*\right]E_\rho + i\omega\left[\omega^2 - \omega_i^2 + i\omega\nu^*\right]E_\varphi\right\},$$

$$v_{i\rho} = \frac{e}{m_i\Delta_1}\left\{-i\omega\left[\omega^2 - \omega_e^2 + i\omega\nu^*\right]E_\rho + \omega_e\left[\mu\left(\omega^2 - \omega_e^2\right) + i\omega\nu^*\right]E_\varphi\right\},$$

$$v_{i\varphi} = -\frac{e}{m_i\Delta_1}\left\{\omega_e\left[\mu\left(\omega^2 - \omega_e^2\right) + i\omega\nu^*\right]E_\rho + i\omega\left[\omega^2 - \omega_e^2 + i\omega\nu^*\right]E_\varphi\right\},$$

where: $\Delta_1 = \omega^2\omega_e^2(1-\mu)^2 - \left[\omega^2 - \omega_e\omega_i + i\omega\nu^*\right]^2$, $\nu^* = \nu(1+\mu)$

Let us consider equation 2./4/ for several limiting cases

If $\omega \ll \omega_i$, $\omega \nu \ll \omega_e \omega_i$ then

$$\varepsilon_\perp = 1 + \frac{\Omega_e^2}{\omega_e \omega_i}, \quad \varepsilon_2 = \frac{\omega \Omega_e^2}{\omega_e \omega_i^2}, \quad \varepsilon_3 = 1 - \frac{\Omega_e^2}{\omega(\omega + i\nu)}$$
$$K_\perp^2 = \varepsilon_1 K^2 - K_3^2, \quad K_2^2 = \frac{\varepsilon_3}{\varepsilon_1}(\varepsilon_1 K^2 - K_3^2) \qquad 2./5/.$$

Assuming that $K_1^2 R_0^2 \ll 1$, $K_2^2 R_0^2 \gtrsim 1$, $\varkappa^2 R_0^2 \gg 1$, $\Omega_e^2 \gg \omega_e \omega_i$ we obtain the dispersion equation for magneto-hydrodynamic waves in a bounded medium

$$\mathcal{J}_1(K_2 R_0) = 0, \quad K_2^2 R_0^2 = \lambda^2 p \qquad \lambda_p = 3{,}8\ldots, \ldots,$$

from here we obtain

$$V_\phi^2 = \frac{\omega_e \omega_i R_0^2}{\lambda_p^2 (1 + i\gamma) + \frac{\Omega_e^2 R_0^2}{c^2}}, \qquad \gamma = \frac{\nu}{\omega} \qquad 2./6/.$$

If the plasma velocity is small $\left[V_\phi = \frac{\omega}{K_3} \ll c\right]$, the solution to the dispersion equation close to the ion cyclotron frequency is as follows:

$$\omega = \omega_i \left(1 - \frac{\Omega_e^2 \beta^2 \phi}{\omega_e \omega_i} - \frac{c^2 \lambda_p^2 \beta^2 \phi}{2 \omega_e \omega_i R_0^2}\right)$$
$$\Omega_e^2 \gg \omega_e \omega_i, \quad \beta\phi \ll \frac{\omega_i^2}{\Omega_e^2}; \quad \frac{\nu}{\omega_i} \ll \frac{\omega_i - \omega}{\omega_i} \ll 1 \qquad 2./7/.$$

Here, the components of the electric and magnetic fields inside the plasma are as follows:

$$E_z = E_0 \left\{ \frac{J_0(K_1 \rho)}{J_0(K_1 R_0)} - \frac{1}{\varepsilon_3} \frac{J_0(K_3 \rho)}{J_0(K_3 R_0)} \right\}$$

$$H_z = E_0 \left\{ -\frac{iK}{K^3}\left(1 + \frac{2\omega_e \omega_i \alpha}{\Omega_e^2} - \frac{K_1^2}{K^2}\right) \frac{J_0(K_1 \rho)}{J_0(K_1 R_0)} + \frac{iK_3}{\varepsilon_3 K} \frac{J_0(K_3 \rho)}{J_0(K_3 R_0)} \right\},$$

$$H_\varphi = -i \varepsilon_3 E_0 \left\{ \frac{K}{K_1} \cdot \frac{J_1(K_1 \rho)}{J_0(K_1 R_0)} - \frac{K}{\varepsilon_3 K_3} \frac{J_1(K_3 \rho)}{J_0(K_3 R_0)} \right\}$$

$$E_\rho = E_0 \left\{ \frac{i \varepsilon_3 K^2}{K_3 K_1}\left(1 + \frac{2\omega_e \omega_i \alpha}{\Omega_e^2}\right) \frac{J_1(K_1 \rho)}{J_0(K_1 R_0)} + \frac{2iK^2}{K_1^2}\left(1 + \frac{\omega_e \omega_i \alpha}{\Omega_e^2}\right) \frac{J_1(K_3 \rho)}{J_0(K_3 R_0)} \right\} \qquad 2./8/.$$

$$E_\varphi = E_0 \left\{ -\frac{\varepsilon_3 K^2}{K_1 K_3} \frac{J_1(K_1 \rho)}{J_0(K_1 R_0)} - 2\left(1 + \frac{\omega_e \omega_i \alpha}{\Omega_e^2}\right) \frac{K^2}{\varepsilon_3^2 K_1^2} \cdot \frac{J_1(K_3 \rho)}{J_0(K_3 R_0)} \right\}$$

where $\alpha = \frac{\omega - \omega_i}{\omega_i \beta^2 \phi}$

For $\mu = 0$, $\nu = 0$, $(\varepsilon_3 \to -\infty)$ equations 2./4/ and 2./7/ go over into the results of Reference (14).

In the case of very fast waves ($\beta_\phi \to \infty$), dispersion equation 2./4/ breaks down into two equations with a definite type of wave corresponding to each of them

$$\frac{1}{K_1} \frac{J_1(K_1 R_0)}{J_0(K_1 R_0)} \mp \frac{1}{iK} \frac{K_1(\mp iK R_0)}{K_0(\mp iK R_0)} = 0,$$

$$K_2 \frac{J_1(K_2 R_0)}{J_0(K_2 R_0)} \mp \frac{K}{i} \frac{K_1(\mp iK R_0)}{K_0(K_0 \mp iK R_0)} = 0 \qquad 2./9/$$

where

$$K_1^2 = \varepsilon_\perp' K^2 \qquad \varepsilon_\perp = \frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1}$$

$$K_2^2 = \varepsilon_\parallel K^2 \qquad \varepsilon_\parallel = \varepsilon_3$$

Waves with a phase velocity equal to c, cannot be propagated in the plasma cylinder since the condition for radiation at infinity is not satisfied in this case. However, if the plasma cylinder is surrounded by a metal casing, these waves may be propagated.

Here the dispersion equation is as follows:

$$\frac{1}{2} \varepsilon_1 \delta (\beta^2 - 1)(K_2^2 - K_1^2) K_0^2 + K_1 R_0 Q \frac{J_1(K_1 R_0)}{J_0(K_1 R_0)} - K_2 R_0 S \frac{J_1(K_2 R_0)}{J_0(K_2 R_0)} = 0 \qquad 2./10/,$$

where $\beta = \frac{R_{casing}}{R_0}$

The corresponding components of the electric and magnetic fields are as follows:

$$\beta R_0 \geqslant \rho \geqslant R_0$$

$$E_z = const = 0$$

$$H_z = const = H_0$$

$$E_\varphi = -H_\rho = \frac{iH_0}{2K\rho} \left\{ K^2 \rho^2 - \beta^2 K^3 R_0^2 \right\} \qquad 2./10a/.$$

$$E_\rho = H_\varphi = H_{\varphi 0} \cdot \frac{R_0}{\rho}$$

$$H_{\varphi 0} = \frac{\varepsilon_2 \varepsilon_3}{\varepsilon_1} \frac{K_2 H_0}{K_2^2 - K_1^2} \left[\frac{K}{K_1} \frac{J_1(K_1 R_0)}{J_0(K_1 R_0)} - \frac{K}{K_2} \frac{J_1(K_2 R_0)}{J_0(K_2 R_0)} \right]$$

$$R_0 \geqslant \rho \geqslant 0$$

$$E_z = \frac{i\varepsilon_2}{\varepsilon_1}\,\frac{K^2 H_0}{K_2^2-K_1^2}\left\{\frac{J_0(K_1\rho)}{J_0(K_1R_0)} - \frac{J_0(K_2\rho)}{J_0(K_2R_0)}\right\},$$

$$E_\rho = \frac{\varepsilon_2}{\varepsilon_1}\,\frac{K_2 H_0}{K_2^2-K_1^2}\left\{\frac{K_1\mathfrak{A}}{K\delta}\,\frac{J_1(K_1\rho)}{J_0(K_1\rho)} - \frac{K_2 R}{K\delta}\,\frac{J_1(K_2\rho)}{J_0(K_2R_0)}\right\},$$

$$H_\varphi = \frac{\varepsilon_2\varepsilon_3}{\varepsilon_1}\,\frac{K^2 H_0}{K_2^2-K_1^2}\left\{\frac{K}{K_1}\,\frac{J_1(K_1\rho)}{J_0(K_1R_0)} - \frac{K}{K_2}\,\frac{J_1(K_2\rho)}{J_0(K_2R_0)}\right\}$$

$$H_z = \frac{H_0}{K_2^2-K_1^2}\left\{M\cdot\frac{J_0(K_1\rho)}{J_0(K_1R_0)} - N\,\frac{J_0(K_2\rho)}{J_0(K_2R_0)}\right\} \qquad 2./10b/.$$

$$E_\varphi = -\frac{iK^2 H_0}{K_2^2-K_1^2}\left\{\frac{K_1 Q}{K\delta\varepsilon_1}\,\frac{J_1(K_1\rho)}{J_0(K_1R_0)} - \frac{K_2 S}{K\delta\varepsilon_1}\cdot\frac{J_1(K_2\rho)}{J_0(K_2R_0)}\right\},$$

$$H_\rho = -E_\varphi$$

If the frequencies of the waves being propagated in the plasma cylinder are large ($\omega \gg \sqrt{\omega_e \omega_i}$), the ion motion may be neglected and dispersion equation 2./4/ reduces to (15):

$$\frac{\varepsilon_3}{K_1K_2}\cdot\frac{J_1(K_1R_0)J_1(K_2R_0)}{J_0(K_1R_0)J_0(K_2R_0)} + \frac{1}{\tilde{\varkappa}^2}\cdot\frac{K_1^2(\tilde{\varkappa}R_0)}{K_0^2(\tilde{\varkappa}R_0)} - \frac{\varepsilon_2^2\varepsilon_3(K^2+K_3^2)(\varepsilon_3K^2+K_3^2)}{2\varepsilon_1^2K_1^2K_2^2(K_1^2-K_2^2)}\times$$
$$\times\frac{1}{\tilde{\varkappa}}\left[K_1\frac{J_1(K_1R_0)}{J_0(K_1R_0)} - K_2\frac{J_1(K_2R_0)}{J_0(K_2R_0)}\right] + \frac{\varepsilon_3\left[(\varepsilon_1K^2-K_3^2)(\varepsilon_1+1)-\varepsilon_2^2K^2\right]}{2\varepsilon_1K_1^2K_2^2}\times$$
$$\times\frac{1}{\tilde{\varkappa}}\,\frac{K_1(\tilde{\varkappa}R_0)}{K_0(\tilde{\varkappa}R_0)}\times\left[K_1\frac{J_1(K_1R_0)}{J_0(K_1R_0)} + K_2\frac{J_1(K_2R_0)}{J_0(K_2R_0)}\right] \qquad 2./11/,$$

where

$$\varepsilon_1 = 1-\frac{\Omega_e^2}{\omega_e^2-\omega^2} \qquad \varepsilon_2 = -\frac{\Omega_e^2\,\omega_e}{\omega(\omega_e^2-\omega^2)} \qquad \varepsilon_3 = 1-\frac{\Omega_e^2}{\omega^2}$$

Thus, slow and fast waves as well as waves with a phase velocity of C may be propagated in a plasma cylinder located in a magnetic field.

Equations 2./2/, 2./3/ and 2./3a/ express the penetration of the field into the plasma cylinder. They determine the form of the field in the plasma cylinder.

It may be seen from these relationships and numerical calculations that electromagnetic waves penetrate quite deeply into the plasma (see parag.1) if the plasma is dense enough $(\omega_2^2 < \omega_e^2 \ll \Omega_e^2)$.

This is connected with the gyrotropic properties of a plasma cylinder.

From equation 2./3/ it follows that the radial distribution of the field depends on ε_1 and ε_2 as well as on ε_3. Therefore, even when $\varepsilon_3 = 1 - \frac{\Omega_e^2}{\omega^2} < 0$, the field penetrates the plasma.

Let us now discuss the question of the energy obtained by the particles in the high frequency electromagnetic field under conditions similar to these of resonance.

The resonance conditions for dense plasma depend on the density and geometry of the plasma in contrast to the case of free electrons and ions whose cyclotron resonance frequency does not depend on these conditions. This is connected with a displacement of the resonance frequencies and a change in the way the field penetrates the plasma.

Here we shall limit ourselves to the two most interesting cases when $k_3 \to \infty$ and when $k_3 \to 0$. In the first case the phase velocity of the wave $\beta_{ph} \to 0$, and the corresponding oscillation frequency coincides with the frequency for ion or electron cyclotron resonances. When $\omega \to \omega_i$ and assuming that

$$\frac{\nu}{\omega_e} \ll \left| \frac{\omega_i - \omega}{\omega_i} \right| \ll 1 \qquad \frac{\Omega_e^2}{\omega_e \omega_i} \gg 1$$

the electron and ion velocities are equal to

$$v_{e\rho} = \frac{eE_\varphi}{m_i \omega_i} \qquad v_{i\rho} = \frac{eE_\varphi}{m_i \omega_i} \frac{\omega_i}{\omega_i - \omega}$$

$$v_{e\varphi} = -i v_{e\rho} \qquad v_{i\varphi} = -i v_{i\rho} \qquad 2./12/.$$

We see that the velocity of the ion is much greater than that of the electron. Let us note that in the case considered the resonance frequency depends but slightly on the plasma density and the geometry of the cylinder 2./7/. It is the same as the cyclotron velocity for a free ion.

When the following conditions are fulfilled

$$\frac{\nu}{\omega_e} \ll \left| \frac{\omega_e - \omega}{\omega_e} \right| \ll 1, \qquad \frac{\Omega_e^2}{\omega_e^2} \gg 1$$

and $\omega \to \omega_e$, the velocities of the particles are equal to:

$$v_{e\rho} = \frac{eE_\varphi}{m_e(\omega_e - \omega)} \qquad v_{i\rho} = \frac{eE_\varphi}{m_i \omega_i}. \qquad 2./13/.$$

$$v_{e\varphi} = i v_{e\rho} \qquad v_{i\varphi} = i v_{i\rho}$$

In this case the electron velocities are much greater than the ion velocities.

Let us now consider the second limiting case, when $k_3 \to 0$. This case corresponds to the oscillation of the cylinder as a whole. Here, the electromagnetic fields does not depend on z.

Dispersion equation 2./4/ with $k_3 = 0$ breaks down into two equations (see 2./9/). The dielectric constant $\varepsilon_\perp$ corresponding to a purely transverse oscillation is equal to [16]:

$$\varepsilon_\perp = \frac{(\omega_e^2-\omega^2)(\omega^2-\omega_i^2)-\Omega_e^4(1+\mu)^2+2(1+\mu)(\omega^2-\omega_e\omega_i)(\Omega_e^2-i\omega\nu)+\omega^2\nu^2(1+\mu)^2}{(\omega_e^2-\omega^2)(\omega^2-\omega_i^2)+(1+\mu)(\omega^2-\omega_e\omega_i)(\Omega_e^2-2i\omega\nu)+i\omega\nu(1+\mu)^2(\Omega_e^2-i\omega\nu)}$$

With $\nu = 0$ the zeros and poles of $\varepsilon_\perp$ will be located at the following frequencies:

Zeros:

$$\omega_\pm = \left[\tfrac{1}{2}(\omega_e^2+\omega_i^2+2\Omega_e^2) \pm \sqrt{\tfrac{1}{4}(\omega_e^2+\omega_i^2+2\Omega_e^2)^2-(\omega_e\omega_i+\Omega_e^2)^2}\right]^{1/2}$$

$$\omega_+ = \begin{cases} \omega_e\left(1+\frac{\Omega_e^2}{\omega_e^2}\right) & \Omega_e^2 \ll \omega_e^2 \\ \Omega_e + \frac{1}{2}\omega_e & \Omega_e^2 \gg \omega_e^2 \end{cases} \qquad 2./14/a$$

$$\omega_- = \begin{cases} \omega_i\left(1+\frac{\Omega_e^2}{\omega_e\omega_i}\right) & \Omega_e^2 \gg \omega_e^2 \\ \Omega_e - \frac{1}{2}\omega_e & \Omega_e^2 \gg \omega_e^2 \end{cases}$$

Poles: 2./14/a

$$\omega_\pm = \left[\tfrac{1}{2}(\omega_e^2+\Omega_i^2+\Omega_e^2) \pm \tfrac{1}{2}\sqrt{(\omega_e^2+\omega_i^2+\Omega_e^2)^2-4\omega_e\omega_i(\Omega_e^2+\omega_e\omega_i)}\right]^{1/2}$$

$$\omega_+ = \begin{cases} \omega_e\left(1+\frac{1}{2}\frac{\Omega_e^2}{\omega_e^2}\right) & \Omega_e^2 \ll \omega_e^2 \\ \Omega_e\left(1+\frac{1}{2}\frac{\omega_e^2}{\Omega_e^2}\right) & \Omega_e^2 \gg \omega_e^2 \end{cases} \qquad 2./14/b$$

$$\omega_- = \begin{cases} \omega_i\left(1+\frac{1}{2}\frac{\Omega_e^2}{\omega_e\omega_i}\right) & \Omega_e^2 \ll \omega_e\omega_i \\ \sqrt{\omega_e\omega_i}\left(1-\frac{1}{2}\frac{\omega_e^2}{\Omega_e^2}\right) & \Omega_e^2 \gg \omega_e^2 \end{cases}$$

The ion and electron velocities and also the corresponding penetration depths are expressed by the following relations: (E_φ is given on the boundary):

1) $\omega = \omega_i\left(1+\frac{1}{2}\frac{\Omega_e^2}{\omega_e\omega_i}\right)$, $\quad \Omega_e^2 \ll \omega_e\omega_i$, $\quad \nu \ll \omega_e$

$$v_{ep} = \frac{eE_\varphi}{m_i\omega_i}\cdot\frac{i\omega_e\nu-\Omega_e^2-\omega_i^2}{2i\omega_e\nu} \qquad v_{ip} = \frac{eE_\varphi}{m_i\omega_i}\,\frac{i\omega_e}{2\nu}$$

$$v_{e\varphi} = \frac{eE_\varphi}{m_i\omega_i}\cdot\frac{i\omega_i\nu-\Omega_e^2}{2\omega_i\nu} \qquad v_{i\varphi} = \frac{eE_\varphi}{m_i\omega_i}\,\frac{\omega_e}{2\nu} \qquad \delta = \frac{c}{\Omega_e}\left(\frac{2\nu}{\omega_i}\right)^{1/2}$$

2) $\omega = \omega_e\left(1+\frac{1}{2}\frac{\Omega_e^2}{\omega_e^2}\right)$ $\quad \Omega_e^2 \ll \omega_e^2$ $\quad \nu \ll \omega_e$

$$v_{ep} = \frac{ieE_\varphi}{m_i\omega_i}\cdot\frac{\omega_e}{2\nu} \qquad v_{ip} = \mu\frac{eE_\varphi}{m_i\omega_i}\cdot\frac{\Omega_e^2-i\omega_e\nu}{2i\omega_e\nu} \qquad \delta = \frac{c}{\Omega_e}\left(\frac{2\nu}{\omega_e}\right)^{1/2}$$

$$v_{e\varphi} = -\frac{eE_\varphi}{m_i\omega_i}\cdot\frac{\omega_e}{2\nu} \qquad v_{i\varphi} = \mu\frac{eE_\varphi}{m_i\omega_i}\,\frac{\mu\Omega_e^2+i\omega_e\nu}{2\omega_e\nu}$$

3) $\omega^2 = \omega_e \omega_i \left(1 - \frac{\omega_e^2}{\Omega_e^2}\right)$ $\qquad \Omega_e^2 \gg \omega_e^2 \qquad \nu \ll \omega_e$

$$v_{e\rho} = \frac{ieE_\varphi}{m_i\omega_i} \cdot \frac{\sqrt{\omega_e\omega_i}}{\nu}; \qquad v_{i\rho} = \frac{ieE_\varphi}{m_i\omega_i} \cdot \frac{\sqrt{\omega_i\omega_e}}{\nu}$$

$$\delta = \frac{c}{\Omega_e}\left(\frac{\nu^2}{\omega_e\omega_i}\right)^{1/4}$$

$$v_{e\varphi} = -\frac{eE_\varphi}{m_i\omega_i} \cdot \frac{\omega_e}{\nu}; \qquad v_{i\varphi} = \frac{eE_\varphi}{m_i\omega_i} \cdot \frac{\omega_i}{\nu}$$

4) $\omega^2 = \Omega_e^2 + \omega_e^2$ $\qquad \Omega_e^2 \gg \omega_e^2 \qquad \nu \ll \omega_e$

$$v_{e\rho} = -i\frac{eE_\varphi}{m_i\omega_i} \cdot \frac{\omega_e^2}{\nu\Omega_e} \qquad v_{i\rho} = -\frac{ieE_\varphi}{m_i\omega_i} \cdot \frac{\omega_e\omega_i}{\nu\Omega_e}$$

$$\delta = \frac{c}{\omega_e}\left(\frac{\nu}{\Omega_e}\right)^{1/2}$$

$$v_{e\varphi} = \frac{eE_\varphi}{m_i\omega_i} \cdot \frac{\omega_e}{\Omega_e} \cdot \frac{i\Omega_e\nu + \omega_e^2}{\Omega_e\nu} \qquad v_{i\varphi} = \frac{eE_\varphi}{m_i\omega_i} \cdot \frac{\omega_i}{\Omega_e} \cdot \frac{\omega_e\omega_i - i\nu\Omega_e}{\nu\Omega_e}$$

For a given component of the external electric field E_ρ on the boundary of the cylinder the electron and ion velocities and corresponding penetration depths are given by the following expressions:

1) $\omega = \omega_i\left(1 + \frac{\Omega_e^2}{\omega_e\omega_i}\right)$ $\qquad \Omega_e^2 \ll \omega_e\omega_i \qquad \nu \ll \omega_e$

$$v_{e\rho} = -\frac{eE_\rho(\Omega_e^4 + 3i\nu\omega_i\Omega_e^2 - 2\nu^2\omega_i^2}{2m_i\nu\omega_i^2\Omega_e^2}, \qquad v_{i\rho} = \frac{eE_\rho(\Omega_e^2 + 2i\omega_i\nu)}{2m_e\nu\Omega_e^2}$$

$$\delta = \frac{c}{\omega_i}\left(\frac{\Omega_e^2}{2\nu\omega_i}\right)^{1/2}$$

$$v_{e\varphi} = \frac{ieE_\rho(\Omega_e^4 + i\omega_i\nu\Omega_e^2 + 2\nu^2\omega_i^2}{2m_e\omega_e\omega_i\Omega_e^2\nu} \qquad v_{i\varphi} = \frac{eE_\rho(\Omega_e^2 + 2i\omega_i\nu)}{2i\,m_e\nu\Omega_e^2}$$

2) $\omega = \omega_e\left(1 + \frac{\Omega_e^2}{\omega_e^2}\right)$ $\qquad \Omega_e^2 \ll \omega_e^2 \qquad \nu \ll \omega_e$

$$v_{e\rho} = -\frac{eE_\rho(\Omega_e^2 + 2i\omega_i\nu)}{2m_e\Omega_e^2\nu} \qquad v_{i\rho} = \frac{eE_\rho(\Omega_e^4 - i\nu\omega_e\Omega_e^2 + 2\omega_e\omega_i\nu^2)}{2m_i\nu\omega_e\Omega_e^2}$$

$$\delta = \frac{c}{\omega_e}\left(\frac{\Omega_e^2}{2\nu\omega_e}\right)^{1/2}$$

$$v_{e\varphi} = \frac{eE_\rho(\Omega_e^2 + 2i\omega_i\nu)}{2m_e\Omega_e^2\nu} \qquad v_{i\varphi} = \frac{ieE_\rho(\Omega_e^4 + i\nu\omega_e\Omega_e^2 - 2\omega_e\omega_i\nu^2)}{2m_i\nu\omega_e^2\Omega_e^2}$$

3) $\omega = \Omega_e + \frac{1}{2}\omega_e$ $\qquad \omega_e \ll \Omega_e \qquad \nu \ll \omega_e$

$$v_{e\rho} = -\frac{e\omega_e E_\rho}{2m_e\nu\Omega_e}, \qquad v_{i\rho} = \frac{e\omega_e E_\rho}{2m_i\nu\Omega_e}$$

$$\delta = \frac{c}{\Omega_e}\left(\frac{\omega_e}{2\nu}\right)^{1/2}$$

$$v_{e\varphi} = -\frac{ie\omega_e E_\rho}{2m_e\nu\Omega_e}\ \left(\mu \ll \frac{\omega_e}{\Omega_e}\right), \qquad v_{i\varphi} = \frac{ie\omega_e E_\varphi}{2m_i\nu\Omega_e}\ \left(\mu \ll \frac{\omega_e}{\Omega_e}\right)$$

4) $\omega = \Omega_e - \frac{1}{2}\omega_e \qquad \omega_e \ll \Omega_e \qquad \nu \ll \omega_e$

$$v_{ep} = \frac{e\omega_e E_p}{2m_e\nu\Omega_e} \qquad v_{ip} = \frac{e\omega_e E_p}{2m_i\nu\Omega_e} \qquad \delta = \frac{c}{\Omega_e}\left(\frac{\omega_e}{2\nu}\right)^{1/2}$$

$$v_{e\varphi} = -\frac{ie\omega_e E_p}{2m_e v\Omega_e}, \quad \left(\mu \ll \frac{\omega_e}{\Omega_e}\right) \qquad v_{i\varphi} = \frac{ie\omega_e E_p}{2m_i\nu\Omega_e} \quad \left(\mu \ll \frac{\omega_e}{\Omega_a}\right)$$

3. Excitation of waves in plasma

We shall now consider the question of exciting waves in the plasma by means of external currents. Let us begin with the simplest problem of exciting hydromagnetic waves in a fluid of infinite conductivity. The state of the fluid is described by the following equations:

$$\rho\frac{\partial v}{\partial t} + \rho(\vec{v}\vec{\nabla})\vec{v} = -\vec{\nabla}p + \frac{1}{c}[\vec{j}, H]$$

$$\frac{\partial\rho}{\partial t} + \operatorname{div}(\rho\vec{v}) = 0 \qquad 3./1/$$

$$\operatorname{rot}\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}$$

$$\operatorname{div}\vec{H} = 0 \qquad 3./2/,$$

$$\operatorname{rot}\vec{H} = \frac{4\pi}{c}(\vec{j} + \vec{j}_0)$$

where $\vec{j}_0$ is the external current density. Assuming the current to be sufficiently small, we may linearize the set of equations 3./1/. As a result, the following equation, which determines the velocity, is obtained:

$$\frac{\partial^2\vec{v}}{\partial t^2} - s^2\vec{\nabla}(\vec{\nabla}\vec{v}) - \left[\operatorname{rot}\operatorname{rot}[\vec{v}, \vec{V}_A], \vec{V}_A\right] = \frac{1}{\rho_0 c}\left[\vec{H}_0, \frac{\partial\vec{j}_0}{\partial t}\right] \qquad 3./3/,$$

where ρ_0 is the equilibrium density of the fluid, s is the velocity of sound waves and $\vec{V}_A = \vec{H}_0/\sqrt{4\pi\rho_0}$

The variable magnetic field $\vec{h} = \vec{H} - \vec{H}_0$ and the change in density caused by the wave are determined from the following equations:

$$\frac{\partial\vec{h}}{\partial t} = \operatorname{rot}[\vec{v}, \vec{H}_0] \qquad 3./4/.$$

$$\frac{\delta\rho}{\partial t} + \rho_0 \operatorname{div}\vec{v} = 0$$

The change in the total energy of the medium per unit of time is equal to

$$J = \frac{1}{c}\int (\vec{v},[\vec{H}_0,\vec{j}_0])\, d\tau \qquad 3./5/.$$

The Fourier velocity component $\vec{v}(\vec{k},\omega)$ is obtained from the equation

$$\left\{\omega^2-(\vec{k},\vec{V}_A)^2\right\}\vec{v}-\left\{(s^2+V_A^2)\vec{k}-(\vec{k},\vec{V}_A)\vec{V}_A\right\}(\vec{k},\vec{v})+\vec{k}(\vec{k},\vec{V}_A)(\vec{V}_A,\vec{v})= \frac{i\omega}{c\rho_0}[\vec{H}_0,\vec{j}_0] \qquad 3./6/,$$

where $\vec{j}(\vec{k},\omega)$ is the Fourier density component of the external current.

Taking the determinant of equation set 3./6/ as zero, we obtain the dispersion equation for natural oscillations of an infinitely conducting fluid located in a magnetic field.

Finding $\vec{v}$ from equation 6, we can derive the following general equation for the radiation intensity of three types of waves, a hydrodynamic wave and two magneto-acoustical waves:

$$dJ = 8\pi s\frac{V_A^2\omega^2}{c^2}\left\{\left|j_\perp'\left(\frac{\omega}{u_1},\theta,\varphi\right)\right|^2\frac{\cos^2\varphi}{u_1^3}+\frac{u_2^2-s^2\cos^2\theta}{u_2^2-u_3^2}\left|j_\perp\left(\frac{\omega}{u_2},\theta,\varphi\right)\right|^2\frac{\sin^2\varphi}{u_2^3}+\frac{s^2\cos^2\theta-u_3^2}{u_2^2-u_3^2}\left|j_\perp\left(\frac{\omega}{u_3},\theta,\varphi\right)\right|^2\frac{\sin^2\varphi}{u_3^3}\right\}do \qquad 3./7/,$$

where U_1^2, U_2^2 and U_3^2 are the squares of the phase velocities of the hydromagnetic and magneto-acoustical waves

$$u_1^2 = V_A^2\cos^2\theta$$

$$u_{2,3}^2 = \frac{1}{2}\left\{(s^2+V_A^2)\pm\sqrt{(s^2+V_A^2)^2-4s^2V_A\cos^2\theta}\right\}$$

θ is the angle between the direction of wave propagation and the magnetic field; φ is the angle between the planes $(\vec{j}_0,\vec{H}_0)$ and $(\vec{k},\vec{H}_0)$.

The external current is considered to be a harmonic function of time.

For a surface current of $\vec{j}=\vec{j}_s\,\delta(z)e^{-i\omega t}$

the total radiation intensity of the hydromagnetic waves is equal to

$$J_s = \pi\frac{V_A}{c^2}j_s^2 \qquad 3./8/.$$

This quantity does not depend on the current frequency.

For a line current the radiation intensity is equal to

$$\mathcal{J}_e = \frac{\pi}{2} \cdot \frac{\omega}{c^2} j_e^2 \qquad 3./9/.$$

Equations 3./1/ and 3./2/ may be used only to describe low-frequency oscillations, whose frequency is much less than the cyclotron frequency of the ions.

Equation set 2./1/ or the simpler set given below (17) may be used to determine the excitation intensity of the waves whose frequency is near the cyclotron ion frequency

$$\begin{aligned} &\rho \frac{\partial v}{\partial t} = \frac{1}{c}[\vec{J}, H_0] \\ &\vec{E} + \frac{1}{c}[\vec{v}, \vec{H}_0] - \frac{m_i}{\rho e c}[\vec{j}, \vec{H}_0] = \frac{m_i m_e}{\rho e^2} \frac{\partial \vec{j}}{\partial t} \qquad 3./10/. \\ &\mathrm{rot}\ \mathrm{rot}\ \vec{E} = -\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{4\pi}{c}\frac{\partial}{\partial t}(\vec{J} + \vec{j}_0) \end{aligned}$$

This set of equations is already linearized. The second equation stands here for the relation $\vec{E} + \frac{1}{c}[\vec{v}, \vec{H}_0] = 0$ which follows from Ohm's law for an infinitely conducting medium. Collisions are not accounted for in equations 3./10/ for the sake of the simplicity.

It may be shown that the Fourier components of the electric field are determined by the following equations

$$\begin{aligned} E_x = \frac{4\pi i}{\omega \Delta} j_0 \Big\{ &\beta^4 \left[(1 - \varepsilon_i - \varepsilon_c)^2 - \varepsilon_i^2\right]\left[1 - \beta^2 \varepsilon_i \varepsilon_e (1 - n^2)\right] n^2 \sin^2\theta \sin\varphi \cos\varphi + \\ &+ i\beta^2 \varepsilon_i \left[1 - \beta^2 \varepsilon_i \varepsilon_e (1 - n^2 + n^2 \cos^2\theta)\right]\Big\} \qquad 3./11/ \\ E_y = -\frac{4\pi i}{\omega \Delta} j_0 \Big\{ &\beta^4 \left[(1 - \varepsilon_i \varepsilon_e)^2 - \varepsilon_i^2\right]\left[1 - n^2 + n^2 \sin^2\theta \cos^2\varphi - \right. \\ &\left. - \beta^2 \varepsilon_i \varepsilon_e (1 - n^2)(1 - n^2 \sin^2\theta \sin^2\varphi)\right] + \\ &+ \beta^2 (1 - \varepsilon_i \varepsilon_e)\left[1 - \beta^2 \varepsilon_i \varepsilon_e (1 - n^2 + n^2 \cos^2\theta)\right] \end{aligned}$$

where

$$\Delta = A n^4 + B n^2 + c, \quad n = \frac{kc}{\omega}, \quad \beta = \frac{V_A}{c}, \quad \varepsilon_i = \frac{\omega}{\omega_i}, \quad \varepsilon_e = \frac{\omega}{\omega_e}$$

$$A = \beta^4 \left\{ (1 - \beta^2 \varepsilon_i \varepsilon_e)\left[(1 - \varepsilon_i \varepsilon_e)^2 - \varepsilon_i^2\right] - (1 - \varepsilon_i \varepsilon_e - \varepsilon_i^2)\sin^2\theta \right\},$$

$$B=-\beta^2\Big\{2(1-\beta^2\varepsilon_i\varepsilon_e)\big(1-\varepsilon_i\varepsilon_e+\beta^2[(1-\varepsilon_i\varepsilon_e)^2-\varepsilon_i^2]\big)-$$

$$-\left[1+\beta^2(1-\varepsilon_e\varepsilon_i-\varepsilon_i^2)\right]\sin^2\theta\Big\},$$

$$C=(1-\beta^2\varepsilon_i\varepsilon_e)\left\{\beta^4\left[(1-\varepsilon_i\varepsilon_e)^2-\varepsilon_i^2\right]+2\beta^2(1-\varepsilon_i\varepsilon_e)+1\right\}$$

(The directions of the axes are given in Fig.2).

Taking Δ as zero we obtain the refraction indices for waves that can be propagated in the medium (here pressure effects are neglected):

$$n_{1,2}^2=1+\left\{1-\varepsilon_i\varepsilon_e-\frac{1-(2-\beta^2)(1-\varepsilon_i\varepsilon_e-\varepsilon_i^2)}{2(1-\beta^2\varepsilon_i\varepsilon_e)}\sin^2\theta\pm\right.$$

$$\left.\pm\,\varepsilon_i\sqrt{1-\frac{\sin^2\theta}{1-\beta^2\varepsilon_i\varepsilon_e}+\left[\frac{1+\beta^2(1-\varepsilon_i\varepsilon_e-\varepsilon_i^2)}{2\varepsilon_i(1-\beta^2\varepsilon_i\varepsilon_e)}\right]^2\sin^4\theta}\right\}\times \qquad 3./13/.$$

$$\times\,\beta^2\cdot\left\{(1-\varepsilon_i\varepsilon_e)^2-\varepsilon_i^2-\frac{1-\varepsilon_i\varepsilon_e-\varepsilon_i^2}{1-\beta^2\varepsilon_i\varepsilon_e}\sin^2\theta\right\}^{-1}$$

The total radiation intensity dI is determined by equation 3./5/. Substituting E_y in it from equation 3./11/, we obtain the following general equation for dI:

$$\mathcal{J}=(2\pi)^4\frac{\omega^2}{c^3}\,\mathrm{Re}\,i\beta^2\int\Big\{\beta^2\Big[1-\beta^2\varepsilon_i\varepsilon_e-n^2(1-\beta^2\varepsilon_i\varepsilon_e)+$$

$$+n^2(1+\varepsilon_i\varepsilon_e\beta^2)\sin^2\theta\cos^2\varphi-n^4\beta^2\varepsilon_i\varepsilon_e\sin^2\theta\sin^2\varphi\Big]\Big[(1-\varepsilon_i\varepsilon_e)^2-\varepsilon_i^2\Big]+$$

$$+(1-\varepsilon_i\varepsilon_e)(1-\beta^2\varepsilon_i\varepsilon_e)+$$

$$+n^2\beta^2\varepsilon_i\varepsilon_e(1-\varepsilon_i\varepsilon_e\sin^2\theta)\Big\}\left(\frac{1}{n^2-n_1^2}-\frac{1}{n^2-n_2^2}\right)\frac{|\vec{j}_0(\vec{n})|^2}{\sqrt{B^2-4AC}}\,d\vec{n}$$

where $\vec{n}=\frac{\vec{k}c}{\omega}$

Let us consider in more detail the excitation of waves by the surface current

$$\vec{j}_0(\vec{z})=\vec{j}_0\,\delta(z)\,e^{-i\omega t}$$

In this case the intensity of radiation per unit surface is equal to:

$$\mathcal{J}=\frac{\pi j_0^2}{c(n_1+n_2)}\left\{\frac{1}{n_1n_2}\left[1+\frac{1-\varepsilon_i\varepsilon_e}{\beta^2\left[(1-\varepsilon_i\varepsilon_e)^2-\varepsilon_i^2\right]}\right]+1\right\} \qquad 3./14/,$$

where

$$n_{1,2}^2=1+\frac{1}{\beta^2(1-\varepsilon_i\varepsilon_e\mp\varepsilon_i)}$$

If $\xi_i(\xi_e \pm 1) = \frac{1+\beta^2}{\beta^2}$ the resonance condition is satisfied and G tends to infinity. The resonance frequencies for the case of surface current excitation are as follows:

$$\omega = \pm \frac{\omega_e}{2} + \sqrt{\frac{\omega_e^2}{4} + \omega_e \omega_i + \Omega_e^2}$$

where

$$\Omega_e^2 = \frac{4\pi \rho e^2}{m_i m_e}$$

Let us consider several limiting cases. If $\xi_e \ll 1$, then

$$J = \frac{\pi j_0}{c(n_1+n_2)} \left\{ \frac{1}{n_1 n_2} \cdot \frac{1+\beta^2(1-\xi_i^2)-\xi_i\xi_e}{\beta^2(1-\xi_i^2)} + 1 \right\} \qquad 3./15/.$$

In this event if $\xi_i^2 \ll 1$, then

$$n_{1,2}^2 = \frac{1+\beta^2}{\beta^2} \qquad J = \frac{\pi\beta j_0^2}{c\sqrt{1+\beta^2}} \qquad 3./16/.$$

If $\xi_i^2 \sim 1$, then

$$n_1^2 = \frac{1}{\beta^2(1-\xi_i)}, \quad n_2^2 = \frac{1+2\beta^2}{2\beta^2}, \quad J = \frac{\pi\beta j_0^2}{c\sqrt{4\beta^2+2}} \qquad 3./17/.$$

If $\xi_i^2 \gg 1$, then

$$n_1^2 = 1 - \frac{1}{\beta^2\xi_i}, \qquad n_2^2 = 1 + \frac{1}{\beta^2\xi_i}$$

$$J = \frac{\pi j_0^2}{c} \cdot \frac{1}{\sqrt{1-\frac{1}{\beta^2\xi_i}} + \sqrt{1-\frac{1}{\beta^2\xi_e}}} \left\{ 1 + \frac{1-\frac{1}{\beta^2\xi_i}+\frac{\xi_e}{\beta^2\xi_i}}{\sqrt{1-\frac{1}{\beta^4\xi_i^2}}} \right\} \qquad 3./18/.$$

Let us now consider the limiting case $\xi_e \sim 1$ ($\xi_i \gg 1$).

Here $n_1^2 = 1 - \frac{1}{2\beta^2\xi_i}, \quad n_2^2 = \frac{1+\beta^2}{\beta^2}$ and

$$J = \frac{\pi j_0^2}{c} \cdot \frac{1}{\sqrt{1-\frac{1}{2\beta^2\xi_i}} + \sqrt{\frac{1+\beta^2}{\beta^2}}} \left\{ 1 + \frac{1+\frac{1}{2\beta^2}}{\sqrt{\left(1-\frac{1}{2\beta^2\xi_i}\right)\left(\frac{1+\beta^2}{\beta^2}\right)}} \right\} \qquad 3./19/.$$

If finally $\xi_e \gg 1$, then

$$n_1^2 = n_2^2 = 1 - \frac{\Omega_e^2}{\omega^2} \qquad 3./20/.$$

$$J = \frac{\pi j_0^2}{c\sqrt{1-\frac{\Omega_e^2}{\omega^2}}}$$

If the plasma is excited by a harmonic line current, we obtain equation 3./9/ for the intensity of radiation per unit in length.

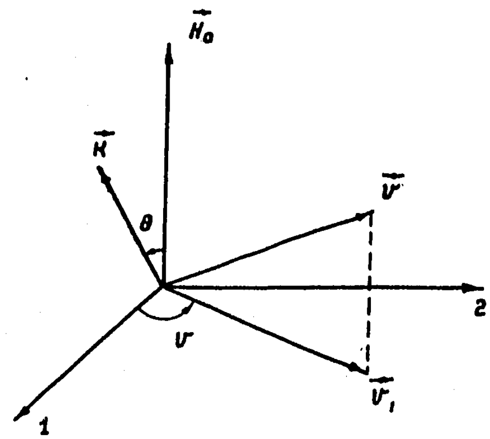

Fig. 1

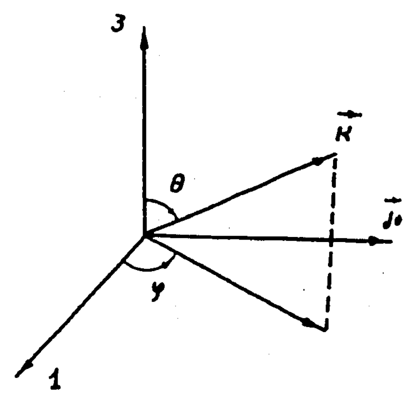

Fig 2

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